COOLING OF A PLANE SLAB OF AN ABSORBING GRAY MEDIUM WITH SIMULTANEOUS CONDUCTION AND RADIATION HEAT TRANSFER

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A method for calculating transient heat transfer by radiation and conduction in a slab of a gray absorbing medium is discussed. The results are given from calculations of the cooling of a slab having a transparent upper boundary and a diffusely reflecting lower boundary in contact with an opaque material.

The calculation of the cooling of a radiatively semitransparent material having an initially high temperature (as in the case of, e.g., molten glass) entails the solution of the transient problem of simultaneous heat transfer by heat conduction and radiation. The calculation of transient mixed heat transfer has been undertaken in a number of papers, the authors of which adopted two main approaches; some [1, 2] linearize the problem, i.e., assume a priori the presence of a small differential temperature, while others [3-5] use iterative methods, the convergence of which is governed by the choice of zeroth approximation. Iterative methods, as a rule, entail a large volume of computational operations, particularly when one takes into consideration the selective nature of absorption in the medium and the temperature dependence of the thermophysical characteristics.

In this paper we partition the main layer (slab) into a number of sublayers and introduce an average temperature for each sublayer to reduce the problem to the solution of a system of nonlinear first-order ordinary differential equations describing the time variation of the average temperatures.

We propose to calculate the cooling of a slab whose upper boundary is transparent to radiation. At the lower boundary, which is diffusely reflecting, we have radiation and conduction heat transfer with an opaque thermally conducting material (Fig. 1).

Simultaneous radiation and conduction heat transfer in a nondissipative gray slab is described by the system of equations [6]

$$\cos\theta \frac{\partial I^{\pm}}{\partial y} = \pm kI_b \mp kI^{\pm}, \tag{1}$$

$$c\rho \frac{\partial T}{\partial t} = \frac{\partial}{\partial y} \lambda_s \frac{\partial T}{\partial y} - 2\pi \int\limits_0^{\pi/2} \cos\theta \frac{\partial}{\partial y} (I^+ - I^-) \sin\theta d\theta \tag{2}$$

with the boundary conditions

$$I^- = I_b(T_a), \quad y = H; \tag{3}$$

$$I^+ = \varepsilon I_b(T_0) + \frac{1-\varepsilon}{\pi} Q_r^-, \; y = 0; \tag{4}$$

$$\frac{\partial T}{\partial y} = 0, \quad y = H; \tag{5}$$

$$T = T_0, \quad \lambda_s \frac{\partial T}{\partial y} + \varepsilon Q_r^- - \varepsilon\pi I_b(T) = \lambda_0 \frac{\partial T_0}{\partial y}, \; y = 0. \tag{6}$$

We thus arrive at the problem of determining functions $T(y, t)$ and $I^+(y, \theta, t)$ satisfying the system (1), (2), the boundary conditions (3)-(6), and a certain given initial temperature distribution $T(y, 0)$.

Partitioning the slab into M auxiliary sublayers and averaging the energy-balance equation (2) over the thickness of the i-th sublayer, we obtain the system of equations

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$$h_i c\rho \frac{dT_i^*}{dt} = \lambda_s \frac{\partial T_i}{\partial y}\Big|_0^{h_i} - 2\pi \int_0^{\pi/2} (I_i^+(h_i, \theta, t) - I_i^+(0, \theta, t) - I_i^-(h_i, \theta, t) + I_i^-(0, \theta, t)) \cos\theta \sin\theta d\theta, \quad (7)$$

in which

$$T_i^*(t) = \frac{1}{h_i} \int_0^{h_i} T_i(y, t)\, dy.$$

An expression for $I_i^+(h_i, \theta, t)$ and $I_i^-(0, \theta, t)$ can be obtained, because Eq. (1) admits a solution in explicit form.

We next assume that the absorption coefficient $k(T_i)$ can be represented by a Taylor series about T_i^*. Up to second-order terms, the expression for the radiation intensity at the boundary of the i-th sublayer acquires the form

$$I_i^+(h_i, \theta, t) = \int_0^{h_i} \left[I_b(T_i^*) + \frac{dI_b}{dT}(T_i^*)\, \delta T_i(y') \right] \exp\left(\frac{k(T_i^*)(y' - h_i)}{\cos\theta} \right) \frac{k(T_i^*)}{\cos\theta} dy' + I_i^+(0, \theta, t) \exp\left(-\frac{k(T_i^*) h_i}{\cos\theta} \right). \quad (8)$$

Here $\delta T_i(y) = T_i(y, t) - T_i^*(t)$. An analogous expression is observed for $I_i^-(0, \theta, t)$. Inasmuch as the radiation leaving the sublayer is the entering radiation relative to the next sublayer, we have the recursion relations

$$I_i^+(0, \theta, t) = I_{i+1}^+(h_{i+1}, \theta, t), \quad 1 \leqslant i < M;$$
$$I_i^-(h_i, \theta, t) = I_{i-1}^-(0, \theta, t), \quad 1 < i \leqslant M. \quad (9)$$

Here $I_1^-(h_1, \theta, t)$, $I_M^+(0, \theta, t)$ are determined from the boundary conditions (3) and (4).

The linear approximation of the thickness temperature profile in the sublayer with respect to T_i^* and the sublayer boundary temperature T_{0i} enables us to carry out the integration in an expression of the type (8). The sublayer boundary temperature T_{0i} in this case can be approximately expressed in terms of $T_i(0, t)$, $T_i^*(t)$, $T_{i-1}^*(t)$ on the basis of the boundary condition (6) and the recursion relations

$$T_M(0, t) = T_0(0, t); \; T_{i-1}(0, t) = T_{0i}, \; 1 < i < M.$$

Thus, relations (9) bring us to the system of equations for the average temperatures

$$h_i c\rho \frac{dT_i^*}{dt} = \lambda_s \frac{\partial T_i}{\partial y}\Big|_0^{h_i} - 2\pi \int_0^{\pi/2} (1 - \exp(-\chi_i)) \cos\theta \sin\theta \Bigg[2I_b(T_i^*) - \sum_{l=i+1}^{M} S^+(T_i^*, T_{0l}, \chi_l) \exp\Big(-\sum_{m=i+1}^{l-1} \chi_m\Big) -$$
$$- \sum_{l=1}^{i-1} S^-(T_i^*, T_{0l}, \chi_l) \exp\Big(-\sum_{m=l+1}^{i-1} \chi_m\Big) - (\varepsilon I_b(T_0(0, t)) + \quad (10)$$
$$+ \frac{1-\varepsilon}{\pi} Q_r^-(0, t)) \exp\Big(-\sum_{m=i+1}^{M} \chi_m\Big) - I_b(T_a) \exp\Big(-\sum_{m=1}^{i-1} \chi_m\Big) \Bigg] d\theta,$$

where

$$\chi_l = k(T_l^*) h_l / \cos\theta; \; S^\pm(T, T_1, \tau) = I_b(T) \pm$$
$$\pm \frac{dI_b}{dT}(T)(T_1 - T) - \left[I_b(T) \mp \frac{dI_b}{dT}(T)(T_1 - T) \right] \exp(-\tau) \mp 2 \frac{dI_b}{dT}(T)(T_1 - T)(1 - \exp(-\tau))/\tau.$$

In carrying out the integration in (10) we encounter the well-known tabulated integral exponential functions

$$E_n(x) = \int_0^{\pi/2} \exp(-x/\cos\theta) \cos^{n-2}\theta \sin\theta d\theta.$$

With the application of the difference approximation for the temperature gradient at the boundaries of the sublayer:

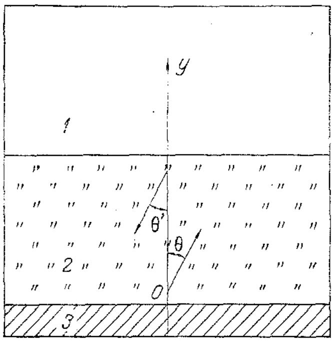

Fig. 1

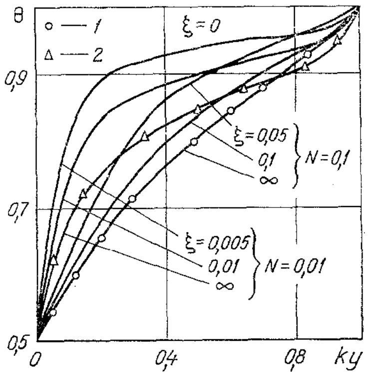

Fig. 2

Fig. 1. Cooling scheme of the radiatively semitransparent slab. 1) Surrounding space; 2) semitransparent medium; 3) opaque material.

Fig. 2. Profiles of the temperature Θ in a slab between isothermal ideally black walls at various times ξ. 1, 2) Exact solution of the steady-state problem for N = 0.1 and 0.01, respectively.

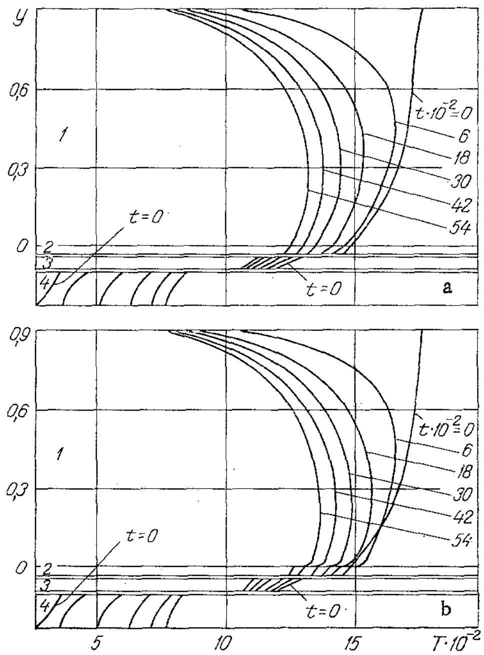

Fig. 3. Temperature distribution T (°K) along the coordinate y (m) in the semitransparent medium (1) and in the opaque material (2-4) at different times t (sec). a) For an ideally black surface y = 0, ε = 1; b) for a reflecting surface y = 0, ε = 0.15.

$$\left.\frac{\partial T_{i-1}}{\partial y}\right|_0 = \left.\frac{\partial T_i}{\partial y}\right|_{h_i} \approx 2\,\frac{T^*_{i-1} - T^*_i}{h_i + h_{i-1}}, \quad 1 < i < M,$$
$$\left.\frac{\partial T_M}{\partial y}\right|_0 \approx 2\,\frac{T^*_M - T_0(0,\ t)}{h_M} \qquad (11)$$

and condition (5), the system (10) comprises a system of M nonlinear first-order ordinary differential equations for $T_i^*(t)$, which can be solved numerically. It is required in this connection to specify the mode of determination of $T_o(0, t)$.

Figure 2 shows the results of a calculation of the temperature stabilization process in a semitransparent slab between isothermal ideally black walls. For the solution we used the Merson modification of the Runge–Kutta numerical procedure with automatic selection of the computing time step on a BESM-6 digital computer. Here the choice of nonuniform partition of the main slab into auxiliary sublayers $\{h_i\}$ was dictated by the condition of obtaining identical transient temperature profiles within 0.5% limits with variation of the thicknesses h_i. For comparison with previous solutions we reduced the results to dimensionless form ($\Theta = T/T_o$, $\xi = tk^2\lambda_s/\rho c$). For various values of the parameter $N = k\lambda_s/4n^2\sigma T_o^3$, characterizing the ratio between conduction and radiation heat transfer, the stabilized solutions are close to the exact solution obtained in [7] for the steady-state case.

When the heat transfer at the lower boundary is stipulated by conditions (6) the system (10) is augmented with a system of differential equations for the average temperatures over the thickness of the auxiliary layers in the opaque material. This system is obtained with the use of the difference approximation for the temperature gradient at the boundaries of the sublayer [by analogy with (11)], the second boundary condition (6), and the heat-transfer conditions at the lower boundary of the opaque material. The temperature $T_o(0, t)$ is determined by extrapolating the temperature profile in the opaque material to its surface.

Figure 3 shows the results of a calculation of the cooling of a glass slab (with a transparent top surface) poured onto a bottom pan consisting of layers of a highly conducting opaque material (the structure of the pan from top to bottom comprises 0.03 m graphite, 0.05 m steel, and 0.13 m pig iron). The distribution observed at the end of pouring of the slab is taken as the initial distribution. The heat transfer between layers of the bottom pan is assumed to be radiative insofar as gaps are created between the layers as a result of their thermal deformation. It is assumed that heat leaves the bottom surface of the pan in accordance with the law of radiation heat transfer with a surrounding medium of temperature T_a. The following values of the parameters are used in the calculations: $H = 0.9$ m; $k = 20\ \text{m}^{-1}$; $n = 1.5$; $\lambda_s = 1.17$ W/m·K; $c = 1.17 \cdot 10^3$ J/kg·K; $\rho = 2.44 \cdot 10^3$ kg/m^3; $T_a = 293$°K.

The curves in Fig. 3a correspond to the case in which the bottom surface ($y = 0$) is ideally black ($\varepsilon = 1$), and the curves in Fig. 3b to the case of a bottom surface comprising reflective foil ($\varepsilon = 0.15$).

A comparison of the curves of Figs. 3a and b shows that the presence of reflection at the bottom surface induces a large temperature drop within a thin surface layer of the medium. The existence of such a drop is attributable to the fact that the opaque material has a higher thermal conductivity than the semitransparent medium ($\lambda_o/\lambda_s \approx 120$). With an increase in the reflection of radiant flux from the surface the ratio of the temperature gradients there in the semitransparent medium and in the opaque material must tend to the inverse ratio of the thermal conductivities of these materials. Outside the thin surface layer, the presence of reflection at the surface affects the temperature field to distances of the order of four mean free paths ($ky = 4$). Here we have a noticeable trend toward a more uniform temperature distribution of the medium and slowing of the cooling process in comparison with the case of an ideally black surface. The temperature difference in this case attains 4%.

In the regions adjacent to the upper (transparent) boundary the temperature distribution is determined mainly by the heat-transfer conditions there. Cooling takes place in such a way as to create at a certain distance from the boundary a region of large temperature gradients, which persist for a long period of time.

NOTATION

y, coordinate; t, time; H, thickness of the slab; $T(y, t)$, temperature; T_a, ambient temperature; $I^+(y, \theta, t)$, intensity of radiation propagating at an acute angle θ with the inward normal to the surface $y = 0$; $I^-(y, \theta, t)$, intensity of radiation propagating at an acute angle θ with the inward normal to the surface $y = H$; $I_b(T) = n^2\sigma T^4/\pi$, total intensity of ideal blackbody radiation; $Q_r^- = 2\pi\int_0^{\pi/2} I^- \cos\theta \sin\theta d\theta$, radiant heat flux density in the direction toward the boundary $y = 0$; T_i, temperature in the i-th auxiliary sublayer; T_i^*, average temperature in the i-th auxiliary sublayer; $I_i^{\pm}$, radiation intensity in the i-th sublayer; h_i, thickness of the i-th sublayer; T_o, temperature of the opaque material; k, absorption coefficient; n, refractive index; c, specific heat; ρ, density; λ_s, thermal conductivity of the slab material; λ_o, thermal conductivity of the opaque material; ε, emissivity of the surface $y = 0$; σ, Stefan–Boltzmann constant; Θ, dimensionless temperature; ξ, dimensionless time; N, conduction–radiation parameter.

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THERMOHYDRAULIC CHARACTERISTICS OF REFRIGERATOR-RADIATOR DEVICES

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Generalized relations are derived for determining the thermal efficiency and the outlet temperature of tubes in radiative heat exchangers with a nonuniform distribution of heat carrier between these tubes.

Finned tubular refrigerator-radiator devices have found broad applications in various branches of engineering [1]. A segment of such a heat exchanger consists of a distributor and a collector connected through a row of tubes with crosspieces between them. Finning of tubes through which the heat carrier flows with heat-emitting crosspieces makes it possible to substantially reduce the metal content of a radiator and improve the energy characteristics of the overall heat-exchanger system.

In this study the thermohydraulic characteristics of radiators and the effect of a nonuniform distribution of heat carrier between their tubes on their thermal efficiency will be considered.

The thermal characteristics of radiative refrigerators with a uniform distribution of heat carrier between tubes have already been studied [2, 3]. In one study [3] the temperature fields over the width of a crosspiece as well as the dependence of the thermal efficiency of a radiator element with finning (Fig. 1) on thermophysical and geometrical parameters of tubes and crosspieces in the case of radiators with black surfaces were determined. Analogous studies were made [2] of diffusely emitting and absorbing gray surfaces.

Elsewhere [4] the results of studies made pertaining to the efficiency of finned tubular radiator elements with tubes at unequal temperatures were reported. The thermal efficiency of a radiator element η_e is defined as the ratio of the amount of heat emitted by it into the surrounding space to the amount of heat emitted by a perfectly black plate of width 2(R + L) at a temperature equal to that of the hotter tube.

The dimensionless temperature field over the width of a crosspiece is described by the equation

$$\frac{d^2\Theta_f}{d\bar{X}^2} = \varepsilon N(\Theta_f^4 - \beta_{f\,inc}) \tag{1}$$

with the boundary conditions

$$X = 0,\ \Theta_f = \Theta_1 = 1,\ X = 2,\ \Theta_f = \Theta_2. \tag{2}$$

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